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Possible Energy Gain at High Latitudes in Southern Hemisphere

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ABSTRACT

It is reasonable to answer to the question: is the rule of thumb, which says that solar collector should be orientated towards the Equator with a tilt equal to latitude, is valid for high latitudes region in Southern Hemisphere? In addition, the question that may arise: how many times is reasonable for adjusting collector tilt angle for Equator facing collectors? A mathematical model was used for estimating the solar radiation on a tilted surface, and to determine the optimum tilt angle and orientation for the solar collector at any latitude. This model was applied for determining optimum tilt angle in the high latitudes zone in the Southern Hemisphere, on a daily basis, as well as for a specific period. The results reveal that changing the tilt angle 12 times in a year maintains approximately the total amount of solar radiation near the maximum value that is found by changing the tilt angle daily to its optimum value. This achieves a yearly gain in solar radiation up to 1.8 times of the case of a horizontal surface while the daily gain reaches 60 times approximately. Moreover, general formulas are proposed for predicting daily optimum tilt angle and optimum tilt angle over any period.

1. Introduction

The performance of a solar collector is highly influenced by its orientation (with respect to the Equator) and its angle of tilt with the horizon (with respect to the ground). This is due to the fact that both the orientation and tilt angle change the solar radiation reaching the surface of the collector. Taking into consideration that designing an installation to yield maximum annual energy helps to minimize the necessary installed capacity and reduce the cost of equipment. To achieve this, the solar collector must be mounted at right angles to the sun's rays.

The best way to collect maximum daily energy is to use tracking systems. A tracker is a mechanical device that follows the direction of the sun on its daily sweep across the sky. The most effective tracking could be achieved by mounting the collector on a two-axis tracker that continuously tracks the sun by the hour and through the seasons. As the trackers are expensive, need energy for their operation this method of tracking is quite cumbersome and inconvenient practically. Thus, the majority of installations are with fixed mountings. Therefore, it is often practicable to orient the solar collector at an optimum tilt angle, B_{opt} and to correct the tilt from time to time. For this purpose, one should be able to determine the optimum slope of the collector at any latitude, for any surface azimuth angle, and on any day or any period of the year. Various schemes have been proposed for optimizing the tilt angle and orientation of solar collectors designed for different geographical latitudes or possible utilization periods. However, as the goal of this work is treat this question regarding the high latitudes region in Southern Hemisphere it is reasonable to restrict ourselves to main available literature concerning this zone directly or indirectly.

In this context, Soulayman [1] proposed a general algorithm for calculating β_{opt} for south facing collector at any latitude from 0^o to 60^o. Soulayman and Sabbagh [2] proposed an algorithm for determining B_{opt} at any latitude, φ, and for any direction (surface azimuth angle, γ). This algorithm could be used for treating B_{opt} in the high latitudes regions. Stanciu, and Stanciu [3] proposed a simple formula for determining the optimum tilt of south facing collector at latitudes from 0^{o} to 80^{o}. Nijegorodov et al. [4] presented 12 equations (one for each month), for determining optimum tilt angle for any location that lies between latitude 60° south to 60° north. Mujahid [5] computed the optimum slope angle for the latitude of 10° north to 50° north and concluded that if the collector adjusted by the seasonally optimum angles, 10% more energy is received compared with the zero slope angle. Calabrò [6] proposed an algorithm to calculate the optimum tilt angle of solar panels by means of global horizontal solar radiation data, provided from Earth-based meteorological stations. This mathematical modeling is based on the maximization of the theoretical expression of the global solar irradiation impinging on an inclined surface, with respect to the slope and orientation of the panel and to the solar hour angle. A set of transcendent equations is obtained, whose solutions give the optimum tilt and orientation of a solar panel. A simulation was carried out using global horizontal solar radiation data from the European Solar Radiation Atlas and some empirical models of diffuse solar radiation. The optimum tilt angle resulted was related to latitude by a linear regression. The standard error of the mean values resulted increased significantly with latitude, suggesting that unreliable values can be provided at high latitudes. So, in the previous studies, no definite value or relation is accepted by all researchers for the optimum tilt angle. Therefore, several attempts were made to determine, or at least to estimate, optimum tilt angle B_{opt} theoretically and experimentally. Soulayman [7] found that at high latitudes the best orientation of the solar collector is The Equator facing case ($\gamma=0^{o}$).

The main objective of this study is to develop a simple and easy way for finding daily, monthly, seasonally, half-yearly and fixed optimum tilt angles for any location in the high latitudes regions and to determine the yearly energy gain. As experimental data concerning the treated question in the studied region are not available to authors, the results of the present study could not be compared with the experimental results. However, the comparison with theoretical results of other researchers will be provided.

2. Materials and Methods

2.1. Main algorithm

Soulayman & Sabbagh [2] found that the optimum tilt angle of a solar collector of any orientation and for any period of time could be obtained by solving the nonlinear algebraic equation:

$$\sum_{n_1}^{n_2} C(n)\left\{\begin{matrix}\left(\frac{\partial A_2}{\partial B}\right)[\sin(\omega_{ss})-\sin(\omega_{sr})]\\ +A_2\left[\cos(\omega_{ss})\left(\frac{\partial \omega_{ss}}{\partial B}\right)-\cos(\omega_{sr})\left(\frac{\partial \omega_{sr}}{\partial B}\right)\right]\\ +\left(\frac{\partial A_1}{\partial B}\right)(\omega_{ss}-\omega_{sr})+A_1\left(\frac{\partial \omega_{ss}}{\partial B}-\frac{\partial \omega_{sr}}{\partial B}\right)\\ -\left(\frac{\partial A_3}{\partial B}\right)[\cos(\omega_{ss})-\cos(\omega_{sr})]\\ +A_3\left[\sin(\omega_{ss})\left(\frac{\partial \omega_{ss}}{\partial B}\right)-\sin(\omega_{sr})\left(\frac{\partial \omega_{sr}}{\partial B}\right)\right]\end{matrix}\right\} = 0 \quad (1)$$

in relation to B where the summation should be ignored when dealing with daily period. In Eq. (1) ω_{ss} and ω_{sr} are the sunset and sunrise hour angle on tilted surface:

$$\omega_{ss} = min\left\{\begin{matrix}arccos[-tan(\delta)\,tan(\varphi)],\\ arccos\left(-\frac{A_1}{A_4}\right)+arcsin\left(\frac{A_3}{A_4}\right)\end{matrix}\right\} \quad (2)$$

$$\omega_{sr} = max\left\{\begin{matrix}-arccos[-tan(\delta)\,tan(\varphi)],\\ -arccos\left(-\frac{A_1}{A_4}\right)+arcsin\left(\frac{A_3}{A_4}\right)\end{matrix}\right\} \quad (3)$$

C(n) is the n^{th} day correction factor for Sun-Earth average distance:

$$C(n) = 1 + 0.034\cos\left(\frac{2\pi n}{365}\right) \quad (4)$$

A_1, A_2, A_3 and A_4 are functions of solar and collector angles:

$$A_1 = \sin(\delta)\left[\begin{matrix}\sin(\varphi)\cos(B)\\ -\sin(B)\cos(\varphi)\cos(\gamma)\end{matrix}\right] \quad (5)$$

$$A_2 = \cos(\delta)\left[\begin{matrix}\cos(\varphi)\cos(B)\\ +\sin(B)\sin(\varphi)\cos(\gamma)\end{matrix}\right] \quad (6)$$

$$A_3 = \cos(\delta)\sin(B)\sin(\gamma) \quad (7)$$

$$A_4 = (A_2^2 + A_3^2)^{0.5} \quad (8)$$

δ is the solar declination angle which could be calculated using the equation of Cooper [41]:

$$\delta = 23.45\sin\left[\frac{2\pi(n+284)}{365}\right] \quad (9)$$

The geometric tilt factor R_b, the ratio of beam radiation on the optimum tilted surface to that on a horizontal surface at any time or period of time, can be calculated exactly in the case of extraterrestrial radiation by appropriate use of solar incidence angle on tilted surface and on the horizon. A similar factor, R_b, could be introduced to express the solar energy gain on the optimum tilted surface to that tilted at angle equal to the latitude. For $B = 0^{o}$, $B = B_{opt}$, $B = \varphi$ daily solar radiation is:

$$H_o(n, L, B, G) = \left(12 \times \frac{3600\,G_{sc}}{\pi}\right)C(n) \times \left\{\begin{matrix}A_2[\sin(\omega_{ss})-\sin(\omega_{sr})]+\\ A_1(\omega_{ss}-\omega_{sr})-\\ A_3[\cos(\omega_{ss})-\cos(\omega_{sr})]\end{matrix}\right\}\frac{J}{m^2} \quad (10)$$

G_{sc} being the solar constant; G_{sc} = 1367 W/m^2.

2.2. General formulae for optimum tilt determination

When applying the algorithms proposed by Soulayman and Sabbagh [8] and Soulayman and Hammoud [9] on the high latitude zones at Southern Hemisphere and analyzing the obtained results for each latitude as a function of δ, it was found that $B_{opt,d}$ can be calculated effectively using the following equation:

$$B_{opt,d} = a\,\delta^2 + b\,\delta + c \tag{11}$$

where "a", "b" and "c" are functions of latitude φ. These functions are of the following form:

$$a = 0.0000027\,\varphi^2 + 0.0001\,\varphi + 0.01247 \tag{12}$$

$$b = 0.00007328\,\varphi^2 + 0.0036486\varphi - 1.48819 \tag{13}$$

$$c = 0.0000026783\,\varphi^2 + 1.01971\,\varphi + 0.0104399 \tag{14}$$

El-Kassaby [10] proposed a formula for determining Bopt,d at latitudes up to 60o in Northern Hemisphere (NH) with verifying the applicability of his formula during the period starting from 22/9 to 21/3 but his formula suffers from uncertainty during period starting from 22/3 to 21/9 (see [1] for more details). Skeiker [11] presented his study aiming to develop an analytical procedure to obtain formula for determining Bopt,d for any chosen day at any latitude in NH but he repeated the same formula of El- Kassaby [10] (see Soulayman and Sabbagh [2] for more details). So, no need to compare the results of the present work with those of El-Kassaby [10] and Skeiker [11]. Figure 1 shows the results of applying Eq. (10) in determining $B_{opt,d}$ for 60°S latitudes.

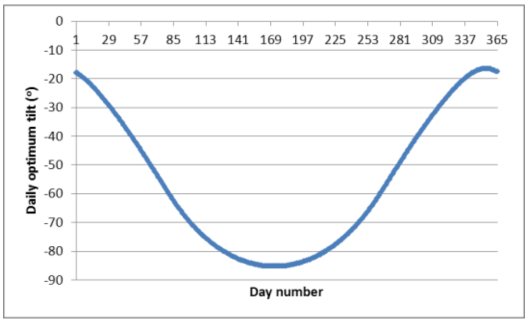

Figure 1. Daily optimum tilt for φ=60°S.

When applying Eq. (10), that of Stanciu, and Stanciu [3], and the algorithm of Soulayman and Sabbagh [8] in calculating $B_{opt,d}$ for φ =50° the obtained results are given in Table 1. It is seen from Table 1 that regarding to the precise results of algorithm of Soulayman and Sabbagh [8], a) For Southern Hemisphere, the equation of Stanciu, and Stanciu [3] underestimates $B_{opt,d}$ remarkably during the period starting from 22/9 to 21/3 and overestimates slightly $B_{opt,d}$ during the period starting from 22/3 to 21/9 with an absolute deviation < 5°; and b) Eq. (10) gives the results of the algorithm with an absolute deviation < 1°; So Eq. (10) could be applied with a very good accuracy with regard to the algorithm of Soulayman and Sabbagh [8] while the equation in [3] gives an acceptable evaluation of daily optimum tilt angle during half a year only.

Table 1. A comparison between different approaches in calculating $B_{opt,d}$(°).

Date	δ(°)	[3]	[8]	Eq.(10)
		φ=-50°		
1/1	-23.01	-27.0	-8	-8.6
15/1	-21.27	-28.7	-12.7	-12.2
1/2	-17.52	-32.5	-20.8	-19.9
15/2	-13.29	-36.7	-28.6	-28.0
1/3	-8.29	-41.7	-37	-37
15/3	-6.76	-43.2	-45.7	-45.6
1/4	4.02	-54.0	-56.1	-56.0
15/4	9.41	-59.4	-63.2	-63.0
1/5	14.90	-64.9	-69.4	-69.3
15/5	18.79	-68.8	-73.3	-73.2
1/6	22.04	-72.0	-76.3	-76.2
15/6	23.31	-73.3	-77.4	-77.3
1/7	23.12	-73.1	-77.2	-77.1
15/7	22.80	-72.8	-75.7	-76.8
1/8	17.91	-67.9	-72.3	-72.4
15/8	13.78	-63.8	-68	-68.1
1/9	7.72	-57.7	-60.8	-60.9
15/9	2.22	-52.2	-53.2	-53.5
1/10	-4.22	-45.8	-43.2	-43.8
15/10	-9.60	-40.4	-34.5	-34.7
1/11	-15.36	-34.6	-24.6	-24.1
15/11	-19.15	-30.9	-17.3	-16.6
1/12	-22.11	-27.9	-10.4	-10.5
15/12	-23.34	-26.7	-6.8	-7.8

When integrating Eq. (10) for obtaining optimum tilt angle $B_{opt,p}$ at any period of time one obtains the following formula:

$$B_{opt,p} = \left[\frac{n_2 - n_1}{n_2 - n_1 + 1}\right]\left\{\left(\begin{matrix} c + 549.9a \\ \left[\begin{matrix} 0.5 - \left(\frac{365}{4\pi}\right) \\ \cos\left[\left(\frac{2\pi}{365}\right)(n_2 + n_1 + 568)\right] \\ * \sin\left[\left(\frac{2\pi}{365}\right)(n_2 - n_1)\right] \end{matrix}\right] \end{matrix}\right)\right\}$$

$$+ \left[\frac{23.45\left(\frac{365}{\pi}\right) b}{n_2 - n_1 + 1}\right] \sin\left[\left(\frac{\pi}{365}\right)(n_2 - n_1)\right] * \tag{15}$$

$$\sin\left[\left(\frac{\pi}{365}\right)(n_2 + n_1 + 568)\right]$$

where n_1 and n_2 are the day numbers of the period beginning and ending respectively.

When calculating monthly optimum tilt angle at latitude 60°S using Eq. (14) and comparing the obtained results with those of set of equations in [4] and set of equations in [12] with taking into consideration the latitudes interval of the applicability of the mentioned references it was observed that in the Southern Hemisphere the results of the present work are in a very good agreement with those of [4], and [12] (see Table 2). However, the equations of [4] give a little bit higher values of optimum angle for the months of April, May, and August as compared with those of [12] and present work while the agreement for other months is very good. It could be noted that the methodologies used in [4], [12] and present work are based on the optimization using mathematical techniques without taking into account the localized patterns of solar radiation falling over a particular location (region). So, an assumption of [4] is responsible for this little disagreement. Nijegorodov et al. [4] used mathematical models for calculating the hourly total radiation and then integrated them to obtain the total daily solar radiation. The main assumption of [4] is the clearness index is constant and equal to 0.7. As the clearness index is not generally constant and varies for each area between two values the slight difference could be because of this assumption.

Statistical indicators used in Table 2 are:

a) The mean bias error (MBE) given by:

$$\mathrm{MBE}=(1/N)\sum(Y_i-Y_{i,ref}) \qquad (16)$$

b) The root mean square error (RMSE) given by:

$$\mathrm{RMSE} = [(1/N)\sum(Y_i-Y_{i,ref})^2]^{0.5} \qquad (17)$$

c) The t-statistic is given as:

$$t= [(N-1)\,\mathrm{MBE}^2/(\mathrm{RMSE}^2 - \mathrm{MBE}^2)]^{0.5} \qquad (18)$$

N: number of data pairs; $Y_i-Y_{i,ref}$ being the difference between the i^{th} predicted and the i^{th} referenced data.

Table 2. Monthly optimum tilt at φ=60°S.

	Eq. (14)	[4]	[12]
1	-22.3	-24.4	-23.7
2	-36.6	-41.2	-38.5
3	-54.6	-56	-56.3
4	-69.9	-70	-72.6
5	-78.9	-79.8	-81.9
6	-82.1	-86.2	-85.6
7	-80.7	-83.4	-83.9
8	-74.0	-75.2	-76.6
9	-60.8	-62	-62.6
10	-42.8	-48	-44.0
11	-25.9	-30.8	-27.4
12	-17.6	-18.2	-17.7
MBE		2.41766	1.425014
RMSE		2.98281	2.234652
t		4.58982	2.745675
R^2		0.994	0.999

3. Results and Discussion

Soulayman [7] found that for latitudes which in absolute value are greater than 23.5° (outside of the tropical region) the best orientation of the solar collector is The Equator facing case (γ=0°). Therefore, as the studied zone is laid between 43.45°S and 66.45°S, the calculated results will be restricted to this case. Moreover, as the formula $B_{opt,d} = \varphi-\delta$ proposed in [3] is a coarse approximation as proven in [7], the comparison of the results of this work with those of [3] is meaningless.

When solving Eq. (1) for obtaining daily optimum tilt angle $B_{opt,d}$ at latitudes from -43.45° to -66° it is found that collector orientation adjustment during the period started from 22/3 to 21/9 is more essential than during the other half-year. A similar result is obtained when solving Eq. (1) For obtaining weekly optimum tilt angle $B_{opt,w}$, monthly tilt angle $B_{opt,m}$ (see Figure 2), seasonally tilt angle $B_{opt,s}$ (see Figure 3) and half-yearly (see Figure 4) at latitudes from -40° to -66°. It is easy to conclude from Figure 2 that solar collector tilt adjustment is essential during the period started from 22/3 to 21/9 inclusively. Here it should be mentioned that, when comparing the results of this work, concerning monthly optimum tilt angle at different latitudes, with those of [4] set of equations, it was observed that the results of [4] are in a good agreement with those of this work. However, the equations of [4] give a little bit higher values of optimum angle for the months of April, May, and August as compared with those obtained in this work while the agreement for other months is very good. Therefore, it should be noted that the equations used in this work, as well as those used in [4], were optimized using mathematical techniques without taking into account the localized patterns of solar radiation falling over a particular location (region). So, an assumption of [4] is responsible for this disagreement. Nijegorodov et al. [4] used mathematical models for calculating the hourly total radiation and then integrated them to obtain the total daily solar radiation. The main assumption of [4] is the clearness index is constant and equal to 0.7. As the clearness index is not generally constant and varies for each area between two values the slight difference could be because of this assumption.

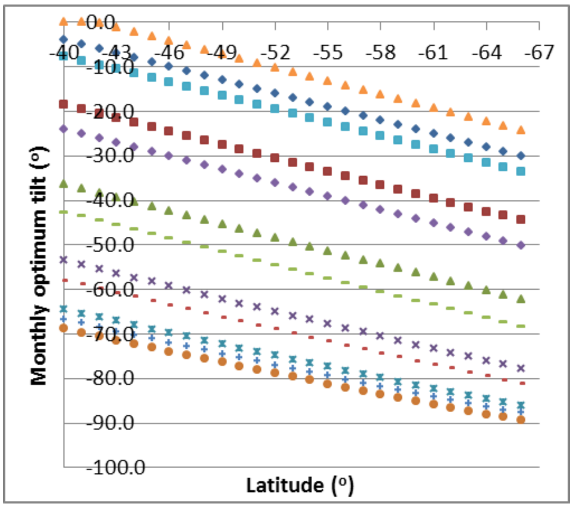

Figure 2. Monthly optimum tilt angle at high latitude zone. ♦, ■, ▲, x, ж, ●, +, -, -, ♦, ■, ▲ stand for January to December respectively.

On the other hand, in order to evaluate the possible solar energy gain using tilt angle adjustment the total yearly extraterrestrial solar radiation at $B = 0^{o}$, $B = B_{opt}$, $B = \varphi$ should be calculated on a daily, monthly, seasonally as well as on a half – yearly and yearly basis. The results are given in Figures 5 to 7 where H_2 stands for solar radiation at $B = B_{opt}$, H_1 stands for solar radiation at $B = \varphi$ and H_0 stands for solar radiation at $B = 0^{o}$. It is seen from Figures 5 to 7 that solar radiation incident on a surface with optimum tilt angle has the maximum values all over the year. When comparing H_2 with H_1 it is seen from Figures 6 and 7 that H_2 is greater than H_1 remarkably over the period started from 22/9 to 21/3 while these values are near to each other during half a year started from 22/3 to 21/9.

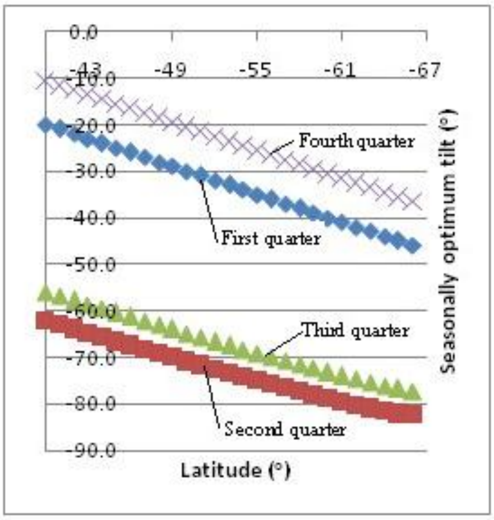

Figure 3. Seasonally optimum tilt.

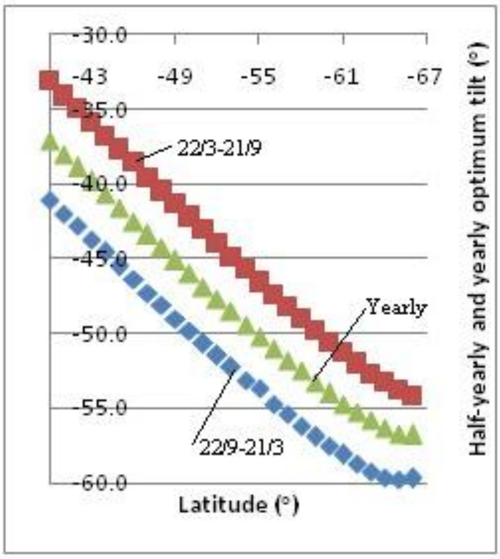

Figure 4. Biannually optimum tilt.

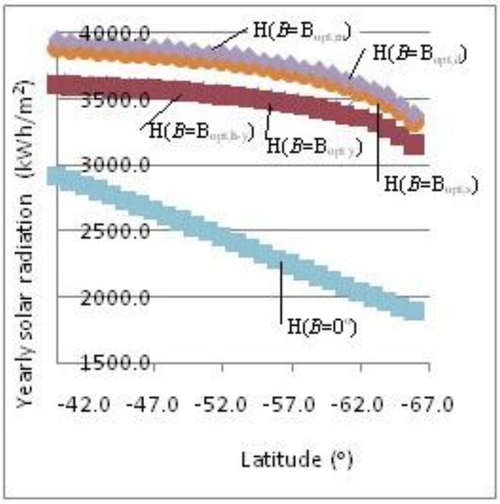

Figure 5. Yearly solar radiation at different latitudes.

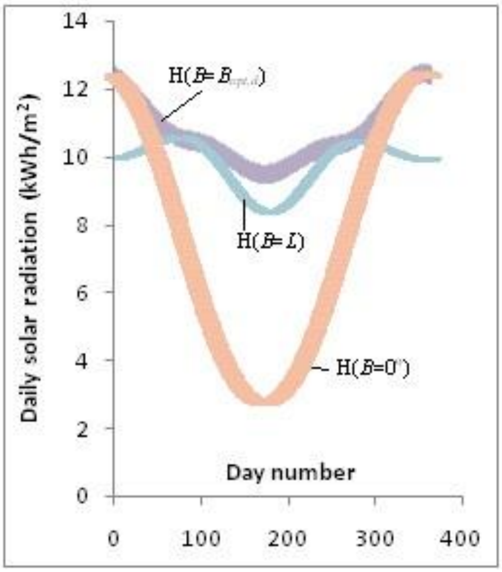

Figure 6. Daily solar radiation incident on surfaces with different tilts for -45^{o} latitude.

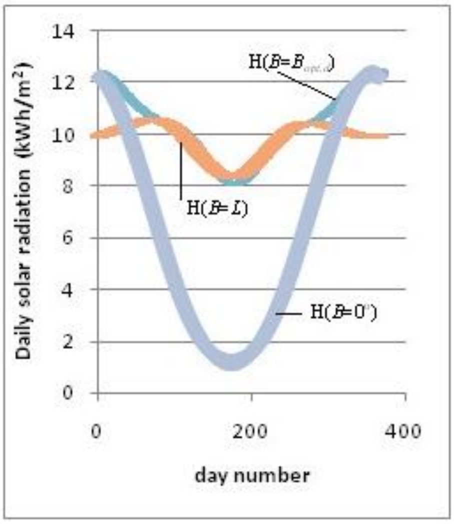

Figure 7. Daily solar radiation incident on surfaces with different tilts for -55^{o} latitude.

By taking the ratio between the values related to surface with optimum tilt to those on a horizontal one and to those tilted by latitude angle for the same period of time, the corresponded tilt factors, $R_b = H_2/H_0$ and $R_{b1} = H_2/H_1$, could be calculated on daily, monthly, seasonally, half-yearly and yearly basis's. These results are given in Figure 8.

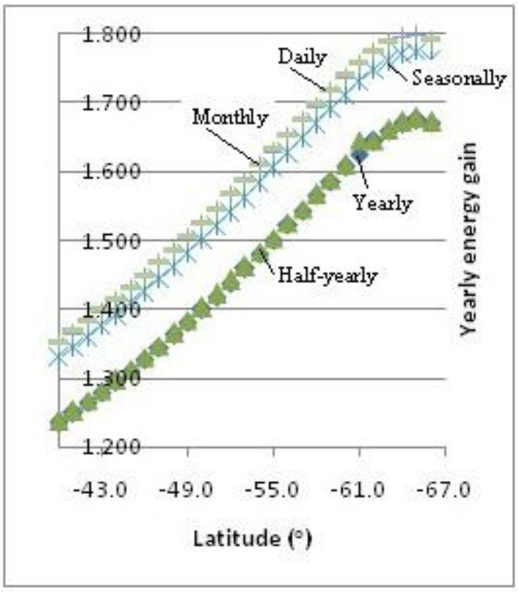

Figure 8. Solar energy gain in relation to the horizontal surface for latitudes [-40^{o}, -66^{o}].

It is found that solar collector tilt adjustment is essential during the period started from 22/3 to 21/9 where the daily solar energy gain can arrive about 60 times of solar radiation on a horizontal plane while this gain is less than 2 times of solar radiation on a horizontal plane during the period started from 22/9 to 21/3. Moreover, it was found that the daily solar energy gain can arrive 1.3 times of solar radiation on latitude tilted plane during the period started from 22/9 to 21/3 while this gain is less than 1.15 times of solar radiation on latitude tilted plane during the period started from 22/3 to 21/9.

4. Conclusions

A mathematical model was applied for determining the optimum tilt angle of the solar collector at any latitude of the interval [-43.45^{o}, -66.45^{o}] in Southern Hemisphere. The optimum tilt angle was computed by searching for the values for which the radiation on the collector surface is a maximum for a particular day or a specific period. For an equator facing flat solar collector:

- It is sufficient to adjust solar collectors tilt angle weekly (once/week) as this adjustment leads to the daily gain approximately.
- It is sufficient to adjust solar collectors tilt angle 12 times (once/month) as this adjustment leads to the daily gain approximately.
- For fixed installations it is practically sufficient to orientate the solar collectors at tilt angle equal to the latitudes during the period started from 22/3 to 21/9 as the losses in the energy gain are less than 0.2.
- The first part of the rule of thumb, which says that solar collector should be orientated towards the Equator is true for high latitudes in Southern hemisphere while the second part, which says that solar collectors should be tilted at angle equal to latitude, is not valid during the period started from 22/9 to 21/3 for high latitudes region.
- General formulae were proposed for determining optimum daily tilt angle and optimum tilt angle for any number of days.
- The applicability of the proposed formulae was verified.

This study will be hopefully expanded to cover Northern high latitudes zone.

Nomenclature

H	Total solar radiation (J/m^{2})
n	Day number in the year
NH	Northern Hemisphere
R	Energy gain factor of daily optimum tilted surface with relation to horizontal surface
R_1	Energy gain factor of monthly optimum tilted surface with relation to horizontal surface
R_2	Energy gain factor of seasonally optimum tilted surface with relation to horizontal one
R_3	Energy gain factor of biannually optimum tilted surface with relation to horizontal one

R_4	Energy gain factor of yearly optimum tilted surface with relation to horizontal surface
R_5	Energy gain factor of latitude tilted surface with relation to horizontal surface
SH	Southern Hemisphere
B	Tilt angle (^{o})
$B_{opt,d}$	Daily optimum tilt angle (^{o})
$B_{opt,b}$	Biannually optimum tilt angle (^{o})
$B_{opt,f}$	Fortnightly optimum tilt angle (^{o})
$B_{opt,m}$	Monthly optimum tilt angle (^{o})
$B_{opt,p}$	Optimum tilt angle over a period (^{o})
$B_{opt,s}$	Seasonally optimum tilt angle (^{o})
$B_{opt,w}$	Weekly optimum tilt angle (^{o})
$B_{opt,y}$	Yearly optimum tilt angle (^{o})
δ	Solar declination angle (^{o})
γ	Collector azimuth angle (^{o})
φ	Latitude angle (^{o}) which is positive for NH and negative for SH
ω_{sr}	Sunrise hour angle on a tilted surface (rad)
ω_{ss}	Sunset hour angle on a tilted surface (rad)

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