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Optimum Tilt Angle and Maximum Possible Solar Energy Gain at High Latitude Zone

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A B S T R A C T

The performance of a solar collector is highly dependent on its tilt angle with the horizon. The variation of tilt angle changes the amount of solar radiation reaching the collector surface. Meanwhile, is the rule of thumb, which says that solar collector should be orientated towards the Equator with a tilt equal to latitude, is valid for high latitudes region? Thus, it is required to determine the optimum tilt for Equator facing collectors. In addition, the question that may arise: how many times is reasonable for adjusting collector tilt angle for Equator facing collectors? A mathematical model was used for estimating the solar radiation on a tilted surface, and to determine the optimum tilt angle and orientation (surface azimuth angle) for the solar collector at any latitude. This model was applied for determining optimum tilt angle in the high latitudes zone in the Southern and Northern Hemispheres, on a daily basis, as well as for a specific period. The optimum angle was computed by searching for the values for which the radiation on the collector surface is a maximum for a particular day or a specific period. The solar radiation on the collector surface of optimum tilt angle, of latitude tilt angle and of null tilt angle was calculated for a particular day or a specific period. The results reveal that changing the tilt angle 12 times in a year (i.e. using the monthly optimum tilt angle) maintains approximately the total amount of solar radiation near the maximum value that is found by changing the tilt angle daily to its optimum value. This achieves a yearly gain in solar radiation up to several times of the case of a horizontal surface depending on the latitude value.

1. Introduction

Solar energy is the most important clean, free and unending renewable energy source which can be utilized in many parts of the world. Utilization of different kinds of solar energy technologies are becoming rapidly widespread. Aligned with recent augmented deployment of these technologies, many studies have been undertaken to enhance the performance of such technologies (Khorasanizadeh et al. [1]). In order to optimize solar isolation on solar collectors, appropriate method to determine solar tilt angles at any given time is essential to increase the efficiencies of the collectors and that of the devices connected to them (Gunerhan and Hepbasli [2]).

A solar system, like any other system, must perform with the highest possible efficiency. This needs the correct design, manufacturing and installation of different components of the system. As for flat solar collectors, the best possible angle of the radiation incidence depends on their orientation and tilt angle; thus determining the proper installation is very vital in order to improve their performance. The optimum tilt angle is influenced by different factors such as the latitude, the clearness index, the air pollution and the distribution of the sunny days throughout the year (Keyanpour-Rad et al. [3]; Siraki and Pillay [4]). According to Benghanem [5], both the orientation and tilt angles have significant effects on the magnitude of the solar radiation reaching the surface of a collector. Therefore, in order to design a solar energy system utilized by flat collectors in a region, performing solar radiation evaluation is essential for attaining suitable information about these factors. A study of this type has been undertaken by Khorasanizadeh et al. [6] for Yazd province of Iran.

Since solar tracking systems have high operation and maintenance costs and are not always applicable, it is often convenient to set the solar collector at a fixed value of an optimum tilt angle (Elminir et al. [7]). However, some algorithms for the minimization of the energy loss generated by the driving actuator were proposed (Seme and Štumberger [8]; Ionită and Alexandru [9]).

Several attempts were made to determine, or at least to estimate, optimum tilt angle β_{opt} theoretically and experimentally. For example, Hottel [10] suggested $\beta_{opt} = \varphi + 20°$, Taybout and Löf [11] proposed $\beta_{opt} = \varphi + (10 \rightarrow 20°)$, Heywood [12] concluded that $\beta_{opt} = \varphi - 10º$, Löf and Taybout [13] proposed $\beta_{opt} = \varphi + (10 \rightarrow 30°)$, Yellott [14] reported $\beta_{opt} = \varphi \pm 20º$, Kern and Harris [15] suggested $\varphi + 10°$, Lunde [16] and Garge [17] suggested $\beta_{opt} = \varphi \pm 15º$, Lewis [18] reported $\beta_{opt} = \varphi \pm 8º$, where φ is latitude of the location and where plus and minus signs are used in winter and summer, respectively. Theoretical models for β_{opt} were suggested by Lewis [18], who considered two different models for β_{opt}.

Some other research works suggested two values for optimum tilt angle, one for summer (rainy season in tropical region) and the other for winter (dry season in tropical region): $\varphi \pm 8°$ (Pavlović et al. [19]; Lewis [18]) and $\varphi \pm 5°$ (Pavlović et al. [19]; Garp and Gupta [20]) ('+' is for winter or dry season, while '−' is for summer or rainy season).

The disagreement among these values may be due to two main reasons:

(1) Firstly, the different methods of calculation that were used for the determination of the optimum slope value of a solar panel;

(2) Secondly, the different empirical models that were considered for the determination of diffuse solar radiance and its link with the amount of global solar radiation.

Kern and Harris [15] explained the calculations related to the optimum slope angle based on the beam radiation. El-Naggar and Chiou [21] carried out an investigation for many regions of the world using different techniques to attain the relation between the optimum slope angle and the latitude. Lewis [18] suggested two theoretical models for determining the optimum angle. In one model, he considered the cloudy and clearness indexes as variables and in another one he used the model of Kern and Harris [15]. El-Kassaby [22] and El-Kassaby and Hassab [23] introduced an analytical equation for finding the optimum slope angle in any latitude and showed that it can be integrated to calculate the optimum slope angle for any period of time. Soulayman [24] by correcting the proposed method of El-Kassaby [22] presented an analytical method for predicting solar collector optimum tilt angle for south facing solar collectors. Morcos [25] developed a mathematical model for calculating the total radiation on a sloped surface and the optimum tilt in Assiut, Egypt. Abdulaziz [26] computed the optimum slope angle for latitudes of 10ºN to 50º N and concluded that if the collector adjusted by the seasonally optimum angles, a gain of 10% in energy is received compared with the zero slope angle.

Nijegorodov et al. [27] presented 12 equations (one for each month), for determining optimum tilt angle for any location that lies between latitude 60°S to 60°N. Hartley et al. [28] calculated the optimum slope angle for Valencia, Spain. Hj Mohd Yakup and Malik [29] computed the monthly optimum slope angle for Brunei, Darussalam. Hussein, et al. [30] used the software TRNSYS and showed that, for Cairo, Egypt, the optimum value of the yearly tilt angle is in the range of 20–30°. Tang and Wu [31] presented an estimation of the optimal tilt angle for maximizing its energy based on the monthly global and diffuse radiation on a horizontal surface.

Ulgen [32] used a mathematical model and determined the monthly, seasonally and yearly optimum slope angles for collectors in Izmir, Turkey. Elminir et al. [7] studied the optimum slope angle theoretically in Helwan, Egypt and compared the results of different mathematical models with experimental results. The results have a little deviation with the experimental results. Gunerhan and Hepbasli [2] calculated the daily optimum slope angle for Izmir, Turkey and compared the results with the results achieved from equations of Nijegorodov et al. [27]. Calabrò [33] proposed an approach of employing sky radiance models for determining optimal tilt angle values of solar collectors using a data archive of the daily global solar radiation collected by the Italian Institute of ENEA in the area of southern-Italy (Sicily).

Chenga et al. [34] analyzed the correlation between the optimal angle for a fixed Building Integrated Photovoltaic (BIPV) system and the latitude of the system's site. Results indicate that an average of 98.6% a system's performance with the optimal angle can be obtained using the latitude angle for the tilted panel. Talebizadeh et al. [35] proposed the genetic algorithm technique for determining the surface azimuth and optimum tilt angles of collectors for installation in Iran. By assuming that the collector surface is always facing toward the equator, Benghanem [5] determined the optimum slope of flat solar collectors in Madinah, Saudi Arabia. He reported that annual optimum tilt angle is approximately equal to latitude, i.e. 23.5°. The optimization of tilt angles was performed using solar radiation data measured for eight big provinces in Turkey Bakirci [36]. Khorasanizadeh et al. [1] developed a horizontal diffuse radiation model for city of Tabass in Iran and determined the optimum tilt angle for south-facing solar surfaces for the fixed monthly, seasonal, semi-yearly and yearly adjustments.

Within the mentioned references a very limited part is related to the high latitude zone ($43.45ºS \le \varphi \le 66.45ºS$ and

43.45°N≤φ≤66.45°N). In this context, Soulayman [24] proposed a general algorithm for calculating β_{opt} for south facing collector. Furthermore, Soulayman and Sabbagh [37] proposed an algorithm which allowed the determination of β_{opt} at any latitude, φ, and for any direction (surface azimuth angle, γ). Stanciu and Stanciu [38] proposed a simple formula for determining the optimum tilt of south facing collector at latitudes from 0° to 80°. Nijegorodov et al. [27] presented 12 equations (one for each month), for determining optimum tilt angle for any location that lies between latitude 60°S to 60°N while El-Kassaby [22] presented these equations for Northern Hemisphere (NH) only. Calabrò [39] proposed such equations for European stations to calculate the optimum tilt angle of solar panels. Therefore, it is of the great importance to be able to determine the optimum slope of the collector at any latitude, for any surface azimuth angle, and on any day or any period of the year in this zone.

The objective of the present study is to present a general algorithm for treating β_{opt} over the latitudes of the high latitude zone (43.45°N≤φ≤66.45°N and 43.45°S≤φ≤66.45°S) and to shed a light on different suggested methods and provided results.

2. Materials and Methods

2.1. Optimum tilt angle

Soulayman and Sabbagh [40], basing on the algorithm, provided by Soulayman [24], gave a nonlinear algebraic equation for daily optimum tilt $\beta_{opt,d}$ determination:

$$\begin{aligned}&\left(\frac{\partial A_2}{\partial \beta}\right)[\sin(\omega_{ss})-\sin(\omega_{sr})]+\\&A_2\left[\cos(\omega_{ss})\left(\frac{\partial \omega_{ss}}{\partial \beta}\right)-\cos(\omega_{sr})\left(\frac{\partial \omega_{sr}}{\partial \beta}\right)\right]\\&+\left(\frac{\partial A_1}{\partial \beta}\right)(\omega_{ss}-\omega_{sr})+\\&A_1\left(\frac{\partial \omega_{ss}}{\partial \beta}-\frac{\partial \omega_{sr}}{\partial \beta}\right)-\left(\frac{\partial A_3}{\partial \beta}\right)\\&.[\cos(\omega_{ss})-\cos(\omega_{sr})]+\\&+A_3\left[\sin(\omega_{ss})\left(\frac{\partial \omega_{ss}}{\partial \beta}\right)-\sin(\omega_{sr})\left(\frac{\partial \omega_{sr}}{\partial \beta}\right)\right]\\&=0\end{aligned} \quad (1)$$

where ω_{ss} (rad) is the sunset hour angle on tilted surface:

$$\omega_{ss}=min\left\{\begin{matrix}arccos[-tan(\delta)\,tan(\varphi)],\\ arccos\left(-\frac{A_1}{A_4}\right)+arcsin\left(\frac{A_3}{A_4}\right)\end{matrix}\right\} \quad (2)$$

ω_{sr} (rad) is the sunrise hour angle on tilted surface:

$$\omega_{sr}=max\left\{\begin{matrix}-arccos[-tan(\delta)\,tan(\varphi)],\\ -arccos\left(-\frac{A_1}{A_4}\right)+arcsin\left(\frac{A_3}{A_4}\right)\end{matrix}\right\} \quad (3)$$

For Equator facing (*EF*) and Pole facing (*PF*) surfaces in both Northern Hemisphere (*NH*) and Southern Hemisphere (*SH*) ω_{ss} = -ω_{sr}.

A_1, A_2, A_3 and A_4 are functions of solar and collector angles:

$$A_1=\sin(\delta)\left[\begin{matrix}\sin(\varphi)\cos(\beta)\\-\sin(\beta)\cos(\varphi)\cos(\gamma)\end{matrix}\right] \quad (4)$$

$$A_2=cos(\delta)\,[cos(\varphi)\,cos(\beta)$$

$$+sin(\beta)\,sin(\varphi)\,cos(\gamma)] \quad (5)$$

$$A_3=cos(\delta)\,sin(\beta)\,sin(\gamma)\,; \quad (6)$$

$$A_4=(A_2^2+A_3^2)^{0.5} \quad (7)$$

δ is the solar declination angle which could be calculated using the equation of Cooper [41]:

$$\delta=23.45sin\left[\frac{2\pi(n+284)}{365}\right] \quad (8)$$

The solution of (1) in relation to the surface tilt β determines $\beta_{opt,d}$.

The optimum tilt over any period of consecutive days, $\beta_{opt,p}$ could be derived directly from the equation:

$$\sum_{n_1}^{n_2}C(n)\left\{\begin{matrix}\left(\frac{\partial A_2}{\partial \beta}\right)[\sin(\omega_{ss})-\sin(\omega_{sr})]+\\A_2\left[\cos(\omega_{ss})\left(\frac{\partial \omega_{ss}}{\partial \beta}\right)-\cos(\omega_{sr})\left(\frac{\partial \omega_{sr}}{\partial \beta}\right)\right]\\+\left(\frac{\partial A_1}{\partial \beta}\right)(\omega_{ss}-\omega_{sr})+A_1\left(\frac{\partial \omega_{ss}}{\partial \beta}-\frac{\partial \omega_{sr}}{\partial \beta}\right)\\-\left(\frac{\partial A_3}{\partial \beta}\right)[\cos(\omega_{ss})-\cos(\omega_{sr})]+\\A_3\left[\sin(\omega_{ss})\left(\frac{\partial \omega_{ss}}{\partial \beta}\right)-\sin(\omega_{sr})\left(\frac{\partial \omega_{sr}}{\partial \beta}\right)\right]\end{matrix}\right\}$$

$$=0 \quad (9)$$

where the summation covers the period in consideration, n_1 is the first day of the period while n_2 is its last day. In (9) *C(n)* is the nth day correction factor for Sun-Earth average distance:

$$C(n) = 1 + 0.034\cos\left(\frac{2\pi n}{365}\right) \quad (10)$$

The analytical solution of Eq. (1), in the case of ω_{ss} tilt angle independence, is:

$$\beta_{opt,d} = \varphi - arctan\left[\frac{\omega_{ss} tan(\delta)}{sin(\omega_{ss})}\right] \quad (11)$$

while this solution, in the case of equation (9), is:

$$\beta_{opt,p} = \varphi$$
$$- arctan\left[\frac{\sum_{n_1}^{n_2} c(n)\omega_{ss}\, sin(\delta)}{\sum_{n_1}^{n_2} c(n) cos(\delta) sin(\omega_{ss})}\right] \quad (12)$$

This situation takes place on *PF* surfaces at equator and for *EF* and *PF* surfaces for other latitudes in *NH* and *SH* for the period from 22/9 to 21/3 in *NH* and for the period from 22/3 to 21/9 in *SH*. For other periods Newton's iteration scheme could be applied for searching the solution of (1), $\beta_{opt,d}$, and of (2), $\beta_{opt,p}$.

2.2. Energy gain

The concept of energy gain is very useful in evaluating optimum tilt application over any period of consecutive days. In this context, it is reasonable to introduce the following functions:

- Energy gain factor of daily optimum tilted surface with relation to horizontal surface:

$$R = \left[\frac{H(\beta=\beta_{opt,d})}{H(\beta=0^o)}\right] \quad (13)$$

- Energy gain factor of monthly optimum tilted surface with relation to horizontal surface:

$$R_1 = \left[\frac{H(\beta=\beta_{opt,m})}{H(\beta=0^o)}\right] \quad (14)$$

- Energy gain factor of seasonally optimum tilted surface with relation to horizontal surface:

$$R_2 = \left[\frac{H(\beta=\beta_{opt,s})}{H(\beta=0^o)}\right] \quad (15)$$

- Energy gain factor of biannually optimum tilted surface with relation to horizontal surface (It is advised here to consider that the solar year starts on 22/3 and ends on 21/3. So, the first half year covers the period from 22/3 to 21/9 while the second half year covers the period from 22/9 to 21/3):

$$R_3 = \left[\frac{H(\beta=\beta_{opt,b})}{H(\beta=0^o)}\right] \quad (16)$$

- Energy gain factor of yearly optimum tilted surface with relation to horizontal surface:

$$R_4 = \left[\frac{H(\beta=\beta_{opt,y})}{H(\beta=0^o)}\right] \quad (17)$$

- Energy gain factor of latitude tilted surface with relation to horizontal surface:

$$R_5 = \left[\frac{H(\beta=\varphi)}{H(\beta=0^o)}\right] \quad (18)$$

Based on the functions in the equations (13) to (19), one can get reliable information regarding the solar receiver installation from practical and economic points of view. So, these functions should be considered for any studied period: daily, weekly, fortnightly, monthly, seasonally, biannually and yearly.

3. Results & Discussion

When applying the equation (1) for calculating the optimum tilt angle over the Earth's surface, located between latitudes 66.45°S and 66.45°N, it is reasonable to divide the mentioned Earth's surface into 3 characteristic zones. The first zone is the tropical between latitudes 23.45°S and 23.45°N. In this zone it was found that, equations (11) and (12) could be applied all over the year with a high accuracy (Soulayman and Sabbagh, [42]). The second zone is the mid-latitude zone between 23.45°N and 43.45°N and between 23.45°S and 43.45°S. In this zone it was found that, equations (11) and (12) could be applied all over the year with an acceptable accuracy (Soulayman and Hammoud, [43]). The third zone is the high-latitude zone between 43.45°N and 66.45°N and between 43.45°S and 66.45°S. In this zone it was found that, the daily and monthly optimum angle, $\beta_{opt,d}$ and $\beta_{opt,m}$, are always positive in the NH and negative in the SH (see Figures 1 and 2). Moreover, $\beta_{opt,d}$ approaches ± 90° when the latitude angle approaches ± 66.45°. On the other hand, the approximate solution, equation (10), leads to large errors with latitude increase (see Figures 3 and 4). The approximate equation (12) leads to similar large errors with latitude increase during the periods of sunset hour angle dependence on the tilt angle.

El-Kassaby [22] used the equation (11) all over the year and Skeiker [44] used the equation (12) all over the year for NH latitudes up to 60°N. Detailed comments on the works of Skeiker [44] and El-Kassaby [22] are given in (Soulayman and Sabbagh, [45]) and (Soulayman and Sabbagh, [37]).

Basing on results of equation (11) El-Kassaby [22] found that, the monthly $\beta_{opt,m}$ in terms of latitude φ could be given by:

$$\beta_{opt,m} = a\,\varphi + b \tag{19}$$

As equation (11) leads to large errors with latitude increase, the proposed coefficients of El-Kassaby [22] lead to large errors also. This equation was used later on by Nijegorodov et al. [27] but with different coefficients *a* and *b* (see Tables 1-4). The calculated monthly optimum tilt angles, $\beta_{opt,m}$, for equator orientated surfaces, using equation (9), could be approximated using equation (19) (see Figure 2). Moreover, it is found that, the calculated seasonally, $\beta_{opt,s}$, biannually, $\beta_{opt,b}$, and yearly $\beta_{opt,y}$ could be approximated using a set of linear equations on latitude φ (see Tables 3 and 4).

Elsayed [46] presented long-term analyses to predict the optimum tilt angle of an absorber plate at any surface azimuth angle γ. The clearness index, K_t, was introduced as a norm to characterize the optimum tilt of south facing collectors at a given point in time when only the global irradiance is known. Elsayed [46] proposed a formula that correlates the monthly optimum tilt angle $\beta_{opt,m}$ in terms of clearness index K_t, latitude φ and the characteristic day number n_{cd} which is the Julian day of the mean day of each month as follows:

$$\begin{aligned}\beta_{opt,m} &= f(\varphi,\ n, K_t, \gamma, \rho) = \\ &(6 - 4.8K_t + 0.86K_t^{0.27}\varphi + 0.0021\,\varphi^2) + \\ &\begin{pmatrix} 31K_t^{0.37} + 0.094K_t^{0.46}\varphi \\ +0.00063K_t^{-0.17}\,\varphi^2 \end{pmatrix} \cos\left[\frac{360(n_{cd}+11.5)}{365}\right]\end{aligned} \tag{20}$$

Table 1. Equation (19) coefficients (Nijegorodov et al. [27]) for NH and SH.

Month	a	b (°)
January	0.89	29
February	0.97	17
March	1.00	4
April	1.00	-10
May	0.93	-24
June	0.87	-34
July	0.89	-30
August	0.97	-17
September	1.00	-2
October	1.00	12
November	0.93	25
December	0.87	34

Table 2. Equation (19) coefficients (El-Kassaby [22]) for NH.

Month	a	b (°)
January	0.82	40.26
February	0.89	50.24
March	0.99	66.23
April	0.93	-13.00
May	0.70	-22.01
June	0.40	-16.01
July	0.40	-12.01
August	0.85	-17.53
September	1.00	-3.01
October	0.96	15.59
November	0.88	29.79
December	0.84	35.21

Table 3. Equation (19) coefficients (this work) for NH.

Month	a	b (°)
January	0.878	31.09
February	0.936	20.49
March	0.992	3.656
April	0.996	-15.14
May	0.960	-29.87
June	1.048	-44.52
July	0.945	-34.00
August	0.980	-20.53
September	0.979	-2.17
October	0.958	15.55
November	0.890	28.86
December	0.860	34.10
Winter (22/12 to 21/3)	0.873	24.77
Spring (22/3 to 21/6)	1	-24.30
Summer (22/6 to 21/9)	0.999	-24.87
Autumn (22/9 to 21/12)	0.875	24.54
1st half (22/3 to 21/9)	1	-24.60
2nd half (22/9 to 21/3)	0.837	24.65
Year	0.916	1.17

Table 4. Equation (19) coefficients (this work) for SH.

Month	a	b (°)
January	1	36.30
February	1	21.50
March	0.998	3.63
April	0.959	-15.02
May	0.891	-28.47
June	0.858	-34.10
July	0.873	-31.48
August	0.935	-20.46
September	0.993	-3.00
October	1	16.04
November	1	32.60
December	0.91	36.90
Winter (22/12 to 21/3)	1	25.40
Spring (22/3 to 21/6)	0.879	-23.75

Summer (22/6 to 21/9)	0.875	-24.27
Autumn (22/9 to 21/12)	1	25.20
1st half (22/3 to 21/9)	0.877	-24.01
2nd half (22/9 to 21/3)	1	25.30
Year	0.875	-1.85

As a consequence of inapplicability of equations (11) and (12) in the high-latitude zone, the algorithm of Skeiker [44] and El-Kassaby [22] should not be applied during the mentioned periods especially in high latitudes of this zone. Tables 5 to 7 show the results of El-Kassaby [22] in comparison with those of this work, Nijegorodov et al. [27] and Soulayman [24] for the latitude 55ºN.

Table 5. The calculated values of $\beta_{opt,m}$ using equation (9) and coefficients of Table 2 for 55ºN.

Month	Table 2	(9)
January	85.61	79.33
February	99.30	72.12
March	120.81	58.50
April	37.88	39.93
May	16.50	23.04
June	6.00	12.67
July	10.00	17.67
August	29.25	33.11
September	52.00	51.56
October	68.40	67.96
November	78.20	77.59
December	81.67	81.29

Table 6. The calculated values of $\beta_{opt,m}$ using coefficients of Tables 1 and 3 for 55ºN.

Month	Table 1	Table 3
January	77.95	79.38
February	70.35	71.97
March	59	58.22
April	45	39.64
May	27.15	22.93
June	13.85	13.12
July	18.95	17.98
August	36.35	33.37
September	53	51.68
October	67	68.24
November	76.15	77.81
December	81.85	81.4

Table 7. The calculated values of $\beta_{opt,m}$ using Soulayman [24] and equation (9) for 55ºN.

Month	[25]	(9)
January	79.2	79.33
February	71.9	72.12
March	58.1	58.50
April	37.5	39.93
May	20.6	23.04
June	9.8	12.67
July	15.8	17.67
August	31.5	33.11
September	51.8	51.56
October	68.2	67.96
November	77.7	77.59
December	81.2	81.29

Even it was expected that, the large deviation of El-Kassaby [22] results will be in the case of months where the sunset hour angle is tilt dependent, it is clear from Table 5 that, the largest deviation occurs on February and March.

On the other hand, an adequate method is proposed for calculating the solar collector optimum tilt. In this context, the precise daily optimum tilt results, calculated for high-latitude zones in NH and SH, were fitted in a polynomial form of declination:

$$\beta_{opt,d} = a\,\delta^2 + b\delta + c \qquad (21)$$

where "*a*", "*b*" and "*c*" are functions of φ. These functions are of the following form:

$$a = 0.0000027(-1)^i\varphi^2 + 0.0001\varphi$$

$$+0.01247(-1)^i \qquad (22)$$

$$b = 0.00007328\varphi^2 + 0.0036486(-1)^i\varphi$$

$$-1.48819 \qquad (23)$$

$$c = 0.0000026783(-1)^i\varphi^2 + 1.01971\varphi$$

$$+0.0104399(-1)^i \qquad (24)$$

where i=0, 1 for SH and NH, respectively. The absolute difference between the results of the equation (14) and precise results, obtained from the equation (8) does not exceed 1.5º.

When integrating the equation (14) for obtaining optimum tilt angle $\beta_{opt,p}$ over any period of time one obtains the following formula:

$$\beta_{opt,p}\,(\;^{o}) = \left[\frac{n_2 - n_1}{n_2 - n_1 + 1}\right]\left\{\begin{array}{c} c + 549.9a0.5 - \\ \left(\frac{365}{4\pi}\right)\cos\left[\left(\frac{2\pi}{365}\right)(n_2 + n_1 + 568)\right] * \\ \sin\left[\left(\frac{2\pi}{365}\right)(n_2 - n_1)\right] \end{array}\right\} +$$

$$\left[\frac{23.45\left(\frac{365}{\pi}\right)b}{n_2 - n_1 + 1}\right] * \sin\left[\left(\frac{\pi}{365}\right)(n_2 - n_1)\right]$$

$$* \sin[(\frac{\pi}{365})(n_2 + n_1 + 568)] \quad (25)$$

where n_1 and n_2 are the day numbers of the period beginning and ending respectively.

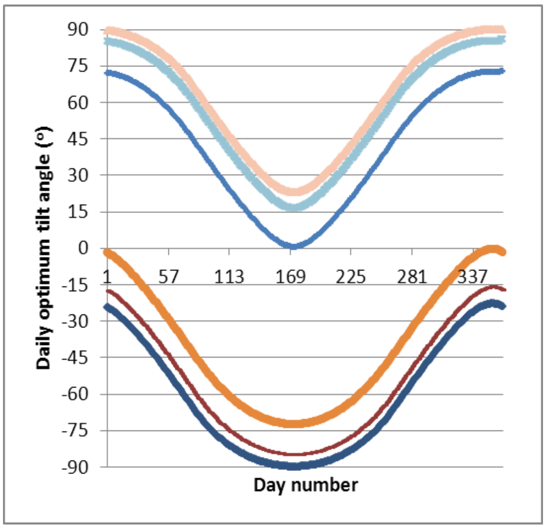

Figure 1. Daily optimum tilt angle at high latitude zone. ● stands for 44°S, - stands for 60°S, ♦ stands for 66.45°S, + stands for 44°N, ж stands for 60°N, and ▲ stands for 66.45°N.

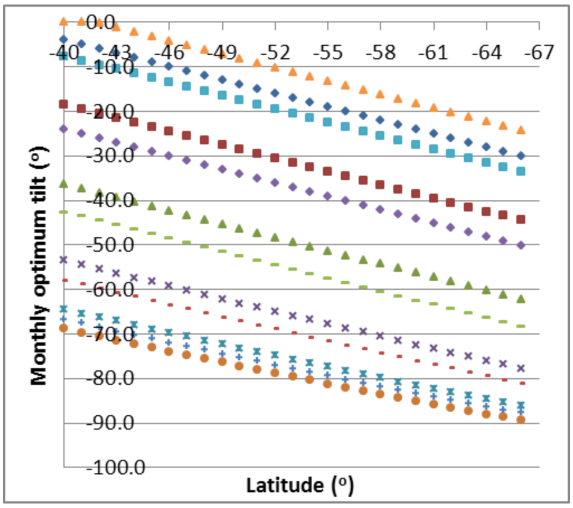

Figure 2. Monthly optimum tilt angle at high latitude zone. ♦, ■, ▲, x, ж, ●, +, -, -, ♦, ■, ▲ stand for January to December respectively.

When applying the equations (9) and (25) in determining $\beta_{opt,p}$ for the latitudes of the mid-latitude zone it was found that, the agreement between their results is of the same order as that for optimum daily tilt angle $\beta_{opt,d}$ in NH and SH. The absolute difference between the results of the equation (25) and precise results, obtained from the equation (9), does not exceed 2.6°.

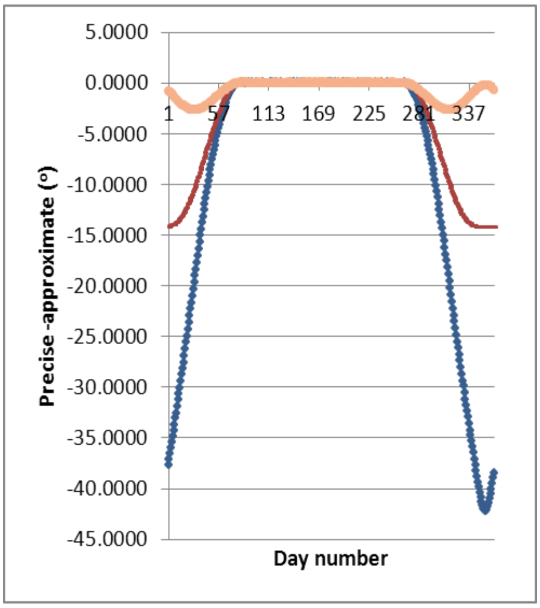

Figure 3. Approximate solution, equation (10), deviation from precise solution of equation (1). ● stands for 43.45°S, - stands for 60°S and ♦ stands for 66.45°S.

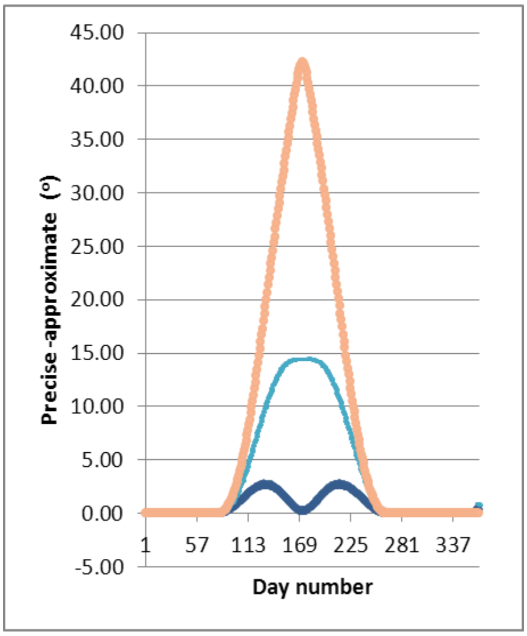

Figure 4. Approximate solution, equation (10), deviation from precise solution of equation (1). ♦ stands for 44°N, - stands for 60°N and ● stands for 66.45°N.

As example, Figure 5 shows the daily energy gain, R, (in times of solar radiation on horizontal plane) of *EF* solar collector for different latitudes in the SH while Figure 6 shows this gain for symmetric latitudes in NH. It is seen from Figures 5 and 6 that, R increases with latitude increase especially during periods where the sunset hour angle is latitude tilt dependent.

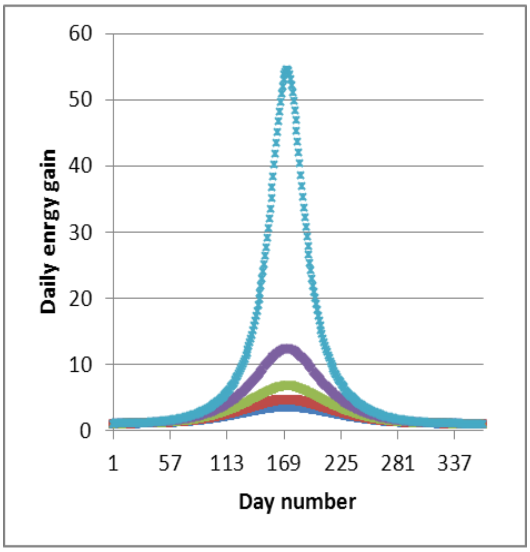

Figure 5. Daily energy gain at latitudes 45°S (♦), 50°S (■), 55°S (▲), 60°S (x) and 65°S (ж).

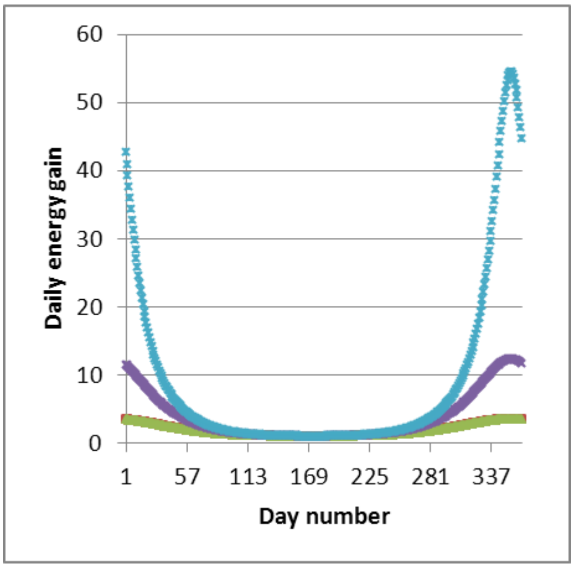

Figure 6. Daily energy gain at latitudes 45°N (♦), 50°N (■), 55°N (▲), 60°N (x) and 65°N (ж).

Of course daily adjustment is efficient than monthly one which in its turn more efficient than seasonally, biannually and yearly adjustments. Therefore, $R>R_1>R_2>R_3>R_4$ and the efficiency of different adjustments can be evaluated with regard to the results of daily adjustment. Thus, by tracing the ratio of R to R_i the judgement with regard to different adjustments could be made. Figures, 7 and 8, show the effectiveness of different adjustments at latitudes 60°N and 50°N respectively. It is seen from Figures 7 and 8 that, it is difficult to distinguish between the results obtained for daily and monthly adjustments which seems suggesting that the energy losses resulting from tilt angle adjustment on monthly basis is negligible. Thus, daily adjustment could be replaced by monthly adjustment without having any significant losses. The seasonal adjustment could be accepted if the losses of 1% in the energy gain are acceptable. The energy gain resulting from biannual adjustment is not effective.

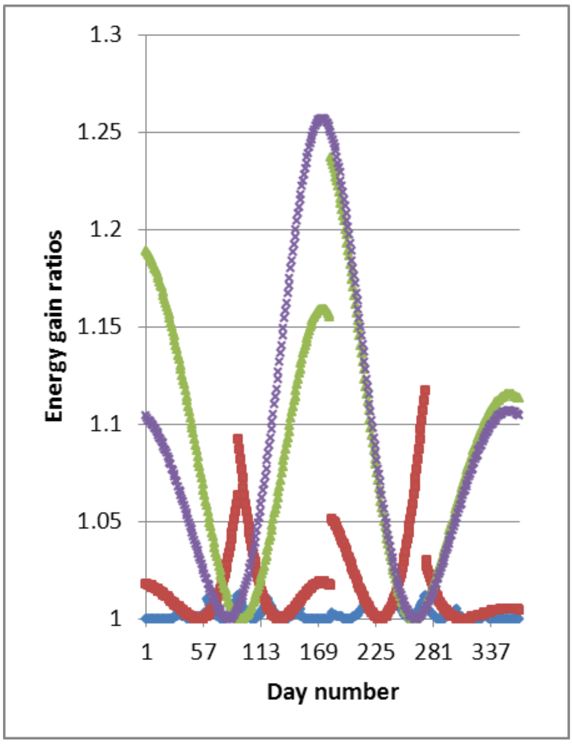

Figure 7. Daily energy gain ratios at latitude 60°N with respect to the monthly (♦), seasonally (■), biannually (▲) and yearly (x).

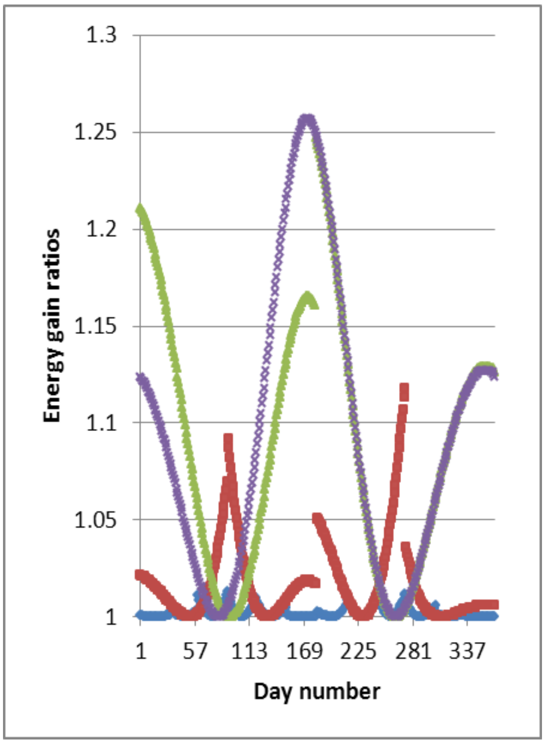

Figure 8. Daily energy gain ratios at latitude 50°N with respect to the monthly (♦), seasonally (■), biannually (▲) and yearly (x).

When calculating the daily energy gain in the case of fixed installation over all the year with yearly optimum tilt angle and latitude tilted angle it was found that, even $\beta_{opt,y}$ is smaller than φ by few degrees (for φ= 43.45°N, $\beta_{opt,y}$ =

40.38º). However, the practical difference between the energy gain results is negligible. This means that the second rule of thumb is valid for high-latitude zone.

Here it should be noted that even optimum orienting solar collector towards Equator could provide an increase in the produced power in relation to other orientations, a lower power production ratio compared with that produced by the tracking systems is envisaged especially during morning and evening. The effect of tilt angle and system orientation is evaluated theoretically and experimentally (Lazaroiu et al., [47]) on the basis of two photovoltaic systems: one fixed and one equipped with a sun tracker. The study verified that the PV module equipped with a single axis sun tracker evidenced an important

4. Conclusions

A mathematical model was applied for determining the optimum tilt angle of the solar collector at any latitude of the interval [(43.45ºS, 66.45ºS) & (43.45ºN, 66.45ºN)] in Southern and Northern Hemispheres. The optimum tilt angle was computed by searching for the values for which the radiation on the collector surface is a maximum for a particular day or a specific period. For an equator facing flat solar collector:

- It is sufficient to adjust solar collectors tilt angle weekly (once/week) as this adjustment leads to the daily gain approximately.
- It is sufficient to adjust solar collectors tilt angle fortnightly (once/two weeks) as this adjustment leads to the daily gain approximately.
- It is sufficient to adjust solar collectors tilt angle 12 times (once/month) as this adjustment leads to the daily gain approximately.
- For fixed installations, it is practically sufficient to orientate the solar collectors at tilt angle equal to the latitude as the energy gain is equivalent to the case of yearly optimum tilt.
- The first part of the rule of thumb, which says that solar collector should be orientated towards the Equator is true for high latitudes in SH and NH and the second part, which says that solar collectors should be tilted at angle equal to latitude, is equivalent to collector yearly adjustment.

General formulae were proposed for determining optimum daily tilt angle and optimum tilt angle for any number of days.

Nomenclature

H	Total solar radiation (J/m^2)
n	Day number in the year
NH	Northern Hemisphere
R	Energy gain factor of daily optimum tilted surface with relation to horizontal surface
R_1	Energy gain factor of monthly optimum tilted surface with relation to horizontal surface
R_2	Energy gain factor of seasonally optimum tilted surface with relation to horizontal one
R_3	Energy gain factor of biannually optimum tilted surface with relation to horizontal one
R_4	Energy gain factor of yearly optimum tilted surface with relation to horizontal surface
R_5	Energy gain factor of latitude tilted surface with relation to horizontal surface
SH	Southern Hemisphere
β	Tilt angle (º)
$\beta_{opt,d}$	Daily optimum tilt angle (º)
$\beta_{opt,b}$	Biannually optimum tilt angle (º)
$\beta_{opt,f}$	Fortnightly optimum tilt angle (º)
$\beta_{opt,m}$	Monthly optimum tilt angle (º)
$\beta_{opt,p}$	Optimum tilt angle over a period (º)
$\beta_{opt,s}$	Seasonally optimum tilt angle (º)
$\beta_{opt,w}$	Weekly optimum tilt angle (º)
$\beta_{opt,y}$	Yearly optimum tilt angle (º)
δ	Solar declination angle (º)
γ	Collector azimuth angle (º)
φ	Latitude angle (º) which is positive for NH and negative for SH
ω_{sr}	Sunrise hour angle on a tilted surface (rad)
ω_{ss}	Sunset hour angle on a tilted surface (rad)

References

[1] Khorasanizadeh, H., Mohammadi, K. and Mostafaeipour, A. (2014). Establishing a diffuse solar radiation model for determining the optimum tilt angle of solar surfaces in Tabass, Iran. Energy Conversion and Management, 78, 805-814.

[2] Gunerhan, H. and Hepbasli, A. (2007). Determination of the optimum tilt angle of solar collectors for building applications. Build. Env., 42(2), 779–783.

[3] Keyanpour-Rad, M., Haghgou, H. R., Bahar, F. and Afshari, E. (2000). Feasibility study of the application of solar heating systems in Iran. Renewable Energy, 20(3), 333-345.

[4] Siraki, A. Gh. and Pillay, P. (2012). Study of optimum tilt angles for solar panels in different latitudes for urban applications. Solar Energy, 86(6), 1920-1928.

[5] Benghanem, M. (2011). Optimization of tilt angle for solar panel: case study for Madinah, Saudi Arabia. Applied Energy, 88(4), 1427–1433.
[6] Khorasanizadeh, H., Mohammadi, K. and Aghaei, A. (2014). The potential and characteristics of solar energy in Yazd province, Iran. Iranica Journal of Energy & Environment, 5(2), 173-183.
[7] Elminir, H. K., Ghitas, A. E., El-Hussainy, F., Hamid, R., Beheary, M. M. and Abdel-Moneim, K. M. (2006). Optimum solar flat-plate collector slope: case study for Helwan, Egypt. Energy Conversion and Management, 47(5), 624–637.
[8] Seme, S. and Štumberger, G. (2011). A novel prediction algorithm for solar angles using solar radiation and differential evolution for dual-axis sun tracking purposes. Solar Energy, 85(11), 2757–2770.
[9] Ionită, M. A. and Alexandru, C. (2012). Dynamic optimization of the tracking system for a pseudo-azimuthal photovoltaic platform. Journal of Renewable and Sustainable Energy,4, Article ID 053117, 15 pages, 2012.
[10] Hottel, H. C. (1954). Performance of flat-plate energy collectors. In: Space Heating with Solar Energy, Proc. Course Symp. Cambridge: MIT Press.
[11] Taybout, R. A. & Löf, G. O. G. (1970). Solar house heating. Nat. Res. J., 10(2), 268-326.
[12] Hyewood, H. (1971). Operating experience with solar water heating. Journal of the Institution of Heating and Ventilation Engineers, 39(6), 63-69.
[13] Löf, G. O. G. and Taybout, R. A. (1973). Cost of house heating with solar energy. Solar Energy, 14(3), 253-278.
[14] Yellott, H. (1973). Utilization of sun and sky radiation for heating cooling of buildings. ASHRAE Journal, 15(12), 31-42.
[15] Kern, J., Harris, I. (1975). On the optimum tilt of a solar collector. Solar Energy, 17(2), 97-102.
[16] Lunde, P. (1980). Solar thermal engineering. New York: Wiley.
[17] Garg, H. (1980). Treatise on solar energy. Fundamentals of solar energy, vol. 1. New York: Wiley.
[18] Lewis G. (1987). Optimum tilt of solar collectors. Solar and Wind Technology, **4**(3), 407-410.
[19] Pavlović, T., Pavlović, Z. L., Pantić, L. and Kostić, J. (2010). Determining optimum tilt angles and orientations of photovoltaic panels in Nis Serbia. Contemporary Materials, 2, 151–156.
[20] Garp, H. P. and Gupta, G. L. (1978). Flat plate collectors: experimental studies and design data for India. International solar energy congress; 16–21 January 1978, New Delhi
[21] El-Naggar, M. M. and Chiou, J. P. (1986). Optimum slope for solar insolation on a flat surface tilted towards the equator in heating season. Solar Energy, 36(5): 471-478.
[22] El-Kassaby, M. M. (1988). Monthly and daily optimum tilt angle for south facing solar collectors; theoretical model, experimental and empirical correlations. Solar & Wind Technology, 5(6), 589-596.
[23] El-Kassaby, M. M. and Hassab, M. H. (1994). Investigation of a variable tilt angle Australian type solar collector. Renewable Energy, 4(3), 327-332.
[24] Soulayman, S. Sh. (1991). On the optimum tilt of solar absorber plates. Renewable Energy, 1(3-4), 551-554.
[25] Morcos, V. H. (1994). Optimum tilt angle and orientation for solar collectors in Assiut, Egypt. Renewable Energy, 4(3), 291-298.
[26] Abdulaziz, M. (1994). Optimum tilt angle for solar collection systems. International Journal of Solar Energy, 14(4), 191-202.
[27] Nijegorodov, N., Devan, K. R. S., Jain, P. K. and Carlsson, S. (1997). Atmospheric transmittance models and an analytical method to predict the optimum slope on an absorber plate. Renewable Energy, 4(5), 529–543.
[28] Hartley, L. E., Martinez-Lozano, J. A., Utrillas, M. P., Tena, F. and Pedro, R. (1999). The optimization of the angle of inclination of a solar collector to maximize the incident solar radiation. Renewable Energy, 6(3), 180-298.
[29] Hj MohdYakup, M. A. B. and Malik, A. Q. (2001). Optimum tilt angle and orientation for solar collector in Brunei Darussalam. Renewable Energy, 24(2), 223-234.
[30] Hussein, H. M. S., Ahmad, G. E. and El-Ghetany, H.H. (2004). Performance evaluation of photovoltaic modules at different tilt angles and orientations. Energy Conversion and Management, 45(15-16), 2441-2452.
[31] Tang, R. and Wu, T. (2004). Optimal tilt-angles for solar collectors used in China. Applied Energy, 79(3), 239-248.
[32] Ulgen, K. (2006). Optimum tilt angle for solar collectors. Energy Sources, Part A: Recovery, Utilization and Environmental Effects, 28(13), 1171-1180.
[33] Calabrò, E. (2009). Determining optimum tilt angles of photovoltaic panels at typical north-tropical latitudes. Journal of Renewable and Sustainable Energy, 1, Article ID 033104.
[34] Chenga, C. L., Jimenez, C. S. S. and Lee, M. (2009). Research of BIPV optimal tilted angle, use of latitude concept for south orientated plans. Renewable Energy, 34(6), 1644-1650.
[35] Talebizadeh, P., Mehrabian, M.A. & Abdolzadeh, M. (2011). Prediction of the optimum slope and surface azimuth angles using the Genetic Algorithm. Energy and Building, 43(11), 2998-3005.
[36] Bakirci, K. (2012). General models for optimum tilt angles of solar panels: Turkey case study. Renewable and Sustainable Energy Reviews, 16, 6149–6159.
[37] Soulayman, S. and Sabbagh, W. (2015). Comment on 'Optimum tilt angle and orientation for solar collectors in Syria' by Skeiker, K. Energy Conversion and Management, 89, 1001-1002.
[38] Stanciu, C. and Stanciu, D. (2014). Optimum tilt angle for flat plate collectors all over the World – A declination dependence formula and comparisons of three solar radiation models. Energy Conversion and Management, 81, 133–143.
[39] Calabrò, E. (2013). An algorithm to determine the optimum tilt angle of a solar panel from global horizontal solar radiation. Journal of Renewable Energy, 2013, Article ID 307547, 12 pages.
[40] Soulayman, S. and Sabbagh, W. (2015b). Optimum tilt angle at tropical region. Int. Journal of Renewable Energy Development, 4 (1), 48-54.

[41] Cooper, P. L. (1969). The absorption of radiation in solar stills. Solar Energy, 12 (3), 333-346.
[42] Soulayman, S. and Sabbagh, W. (2016). General algorithm for optimum tilt angles determination at tropical region. British Journal of Renewable Energy, 1(1), 32-37.
[43] Soulayman, S. and Hammoud, M. (2016). Optimum tilt angle of solar collectors for building applications in mid-latitude zone. Energy Conversion Management, 124, 20-28.
[44] Skeiker, K. (2009). Optimum tilt angle and orientation for solar collectors in Syria. Energy Conversion and Management, 50(9), 2439-2448.
[45] Soulayman, S. and Sabbagh, W. (2014). An algorithm for determining optimum tilt angle and orientation for solar collectors. Journal of Solar Energy Research Updates, 1, 19-30.
[46] Elsayed, M. M. (1989). Optimum orientation of absorber plates. Solar Energy, 42(2), 89–102.
[47] Lazaroiu, G. C., Longo, M., Roscia, M. & Pagano, M. (2015). Comparative analysis of fixed and sun tracking low power PV systems considering energy consumption, Energy Conversion and Management, 92, 143–148.